

# Satellite Image Matching using Kalman Filter and a cross correlation technique

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**Abstract**— In this paper, we propose a method for evaluating the displacement and deformation fields for pair of multi temporal images, one before and one after deformation; based on a matching technique of cross correlation. First, according to the noise present in the images and the long computation time, a denoising based on the Kalman filter and a multi-resolution analysis of efficient optimization scheme are proposed to improve the quality and speed time calculation. Second, an algorithm for matching based on the gray levels cross correlation is introduced to improve the efficiency on matching. Our method was tested on satellite images and the results are encouraging.

**Index Terms**— Matching images, cross-correlation, displacement, deformation, satellite images.

## 1 INTRODUCTION

Optical techniques have opened up a wide field of applications in remote monitoring environments, mobile robotics, control or measurement. The objective of this paper is to develop a method to measure the displacement and deformation fields using the cross-correlation technique in gray levels. The general principle is based on matching images in computer vision. Two steps are required for any measure:

- The matching between the two multi temporal images is to determine a set of homologous pixels corresponding to the same physical point.
- The extent of the displacement field and the calculation of the deformation field between the two images (reference state and deformed state).

Several authors Vincent L., [1] Chambon S., [2] presented a classification of matching methods. The criteria used are generally the nature and dimension of attributes (constituting elements) to match, the field (local or global), the problem to solve, the constraints used, the measurement of similarity measure...

In recent years digital image correlation given by Peters W.H. et al [3] Chu T.C. et al [4] has become increasingly important. Many authors use the correlation Crouzil A. et al [5]. Zhao, F. et al [6] presented a method based on normalized cross-correlation that can effectively manage a pair of images with a big change and Rziza M. et al [7] presented algorithms for estimating disparity, based on the method of estimating a

age and Fookes, C. et al [9] who presented extensions of the stereo matching based on mutual information and a approach with two hierarchical levels in order to increase the robustness of the algorithm. Also Hu Hao et al [10] presented a method of automatic vision to measure the deformation of surfaces. This method is based on image correlation and binocular stereovision. Pablo J. et al [11] proposed using a new method to measure the correlation of the deformation field. Finally, Guérin C. et al [12], use the digital surface models (DSMs) that create a geo-localized map of elevation changes of an urban scene from a set of couple images.

## 2 DESCRIPTION OF THE MATCHING METHOD

Two images are used, the first is known as the reference image, the second is distorted. The steps of our approach are summarized in figure.1

### 2.1 Denoising by the Kalman filter

We will start by denoising step. The KALMAN filter Nemesin V. [13] is a recursive estimator. It has two distinct phases:

A phase of prediction using the estimated state of the previous time to produce an estimate of the current state, the phase of updating the current observation of the state that is used to correct the predicted state in order to obtain a more accurate estimate.

```
For i=1 to nb_iteration do
    Kalmani,j = predictioni,j / (valpredictioni,j - bruiti,j);
    Correctioni,j = gain*predictioni,j + (1-gain) * observationi,j + kalmani,j * (observationi,j - predictioni,j);
    Correction_vari,j = prediction_vari,j * (1 - kalmani,j);
    Prediction_vari,j = correction_vari,j;
    Predictioni,j = Correctioni,j;
End.
```

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dense disparity map by using higher order statistics and robust correlation method. Due to the size of the data, matching can be very large, which is why several methods use a hierarchical of scheme matching like Qin, X. et al [8] who used a pyramid im-

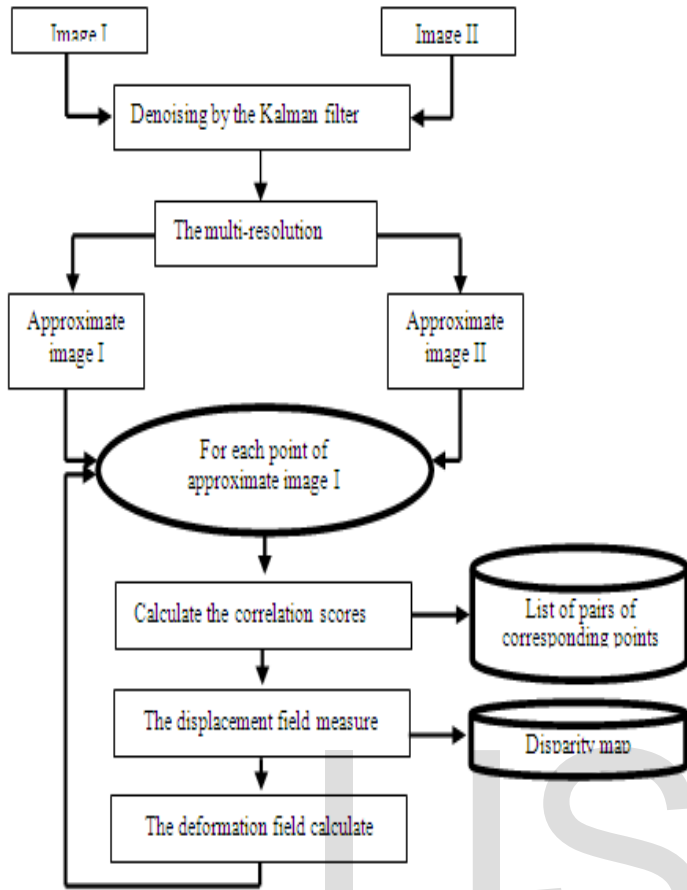


Fig. 1. Different steps of the approach

**2.2 The Multi Resolution**

A multi resolution approach was adapted; the model is based on a multi resolution wavelet decomposition obtained by Daubechies wavelets. Each multi temporal image is decomposed into three levels of resolution; we apply our approach on the level of coarsest resolution. Coupling with a multi resolution analysis allows a computational optimization while approaching the performance of the original method.

**2.3 Image Matching**

The principle of the digital image correlation is shown in fig2. Two images are not taken under the same lighting conditions, or with identical cameras. It is therefore necessary to achieve more than one simple correlation of gray levels.

**2.3.1 Similarity Measure based on Correlation**

The cross-correlation coefficient is used to calculate a normalized correlation factor, which comes down to not comparing the levels of gray but rather the way in which these levels of gray vary. In our approach, we use the MORAVEC measure (robust to noise) and the centred normalized cross correlation measure ZNCC (less expensive in computation time) Lane R A. et al [14].

$$ZNCC = \frac{(f_1 - \bar{f}_1) - (f_2 - \bar{f}_2)}{\|f_1 - \bar{f}_1\| \cdot \|f_2 - \bar{f}_2\|}$$

$$MOR = \frac{2(f_1 - \bar{f}_1) - (f_2 - \bar{f}_2)}{\|f_1 - \bar{f}_1\|^2 + \|f_2 - \bar{f}_2\|^2}$$

$f_i$  with  $i= 1,2$  are vectors containing the pixel gray level of images 1 and 2.

$\bar{f}_i$  with  $i= 1,2$  are the means.

After testing different window size correlations, we chose a window [7x7] with a search box dimension [17x17].

**2.3.2 Correlation algorithm**

$M_i$ : the points of the first image.

$M_j$ : the points of the second image.

F1: window of correlation centred on each point of the first image.

F2: window of correlation sliding on the second image

For each point  $M_i$  of image 1 do

Center the window F1 on the point  $M_i$

Choose an area of research in image2

For each point  $M_j$  of the search ZONE do:

Center the window F2 on the point  $M_j$

Calculate correlation scores associated with the pixel  $M_i$  (Either by measurement or normalized cross-correlation centred or MORAVEC measurement).

Test the coefficient of maximum score

(The point  $M_i$  corresponding to  $M_j$  is one for which the score is greater).

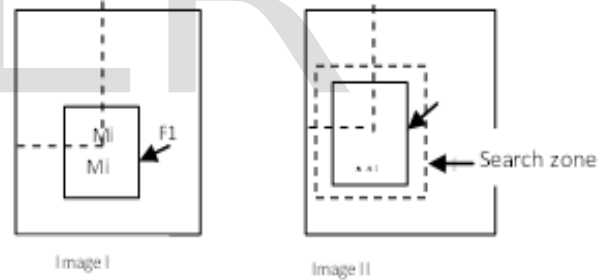


Fig. 2. Search for a match by correlation

The accuracy of displacement measurements depend on the performance of the correlation technique. Given the coordinates of the points in the original image and the coordinates of these physical points of the distorted image, obtained by cross-correlation technique, the displacement field and strain can be calculated.

**2.4 The Measurement of the Displacement Fields**

The deformations are estimated by a post treatment from displacement measure. After matching pairs of homologous points, the displacements are measured by subtracting the coordinates of the initial points and deformed.

$$\begin{cases} x^* = x + \Delta x \\ y^* = y + \Delta y \end{cases} \rightarrow \begin{cases} \Delta x = x^* - x \\ \Delta y = y^* - y \end{cases} \quad (1)$$

Thus, the displacement is represented by

$$\Psi(x) = \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \quad (2)$$

In our method disparity at the point  $x$  is represented by the norm displacement vector at the point  $x$  (Euclidean distances).

### 2.5 The Calculation of the Deformation Fields

Now we describe the deformation model used. The state of deformation can be described by a tensor of deformation defined to any point. We talk of a deformation field. To deduce the deformation at a point, two approaches are used

#### 2.5.1 Averaging of deformation of triangular elements in the vicinity:

The deformations of the considered point correspond to the average value of distortion of the triangular elements. There are two approaches as illustrated in figure. 3. The coefficients assigned to the elements of figure. 3 correspond to the value of the deformation of the triangular element.

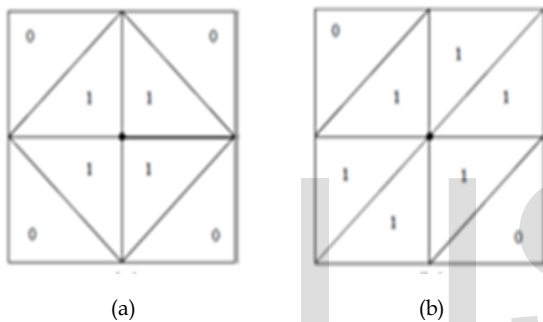


Fig. 3. (a) Localized structure, (b) diagonal structure

#### Principle of calculation of the deformation of a triangular element:

Calculations of deformations are made on triangular elements, assuming that each triangle submitted a homogeneous deformation.

A change of reference must be effected in order to express the coordinates of the triangle points  $(x_0, x_1, x_2)$ . Changes in the reference are effected so that the point whose deformation we are trying to calculate is the origin of referee  $x_0$ ,  $x_1$  is on the  $x$ -axis and  $x_2$  is on vertical axis. A point of initial coordinates  $(x_1, y_1)$ , in a homogeneous deformation will be coordinated  $(x_1^*, y_1^*)$ .

$[F]$  is a second order tensor called deformation gradient such that:

$$[F] = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \quad (3)$$

Components  $(x_1^*, y_1^*)$  and  $(x_2^*, y_2^*)$  can be written as follows:

$$\begin{cases} x_1^* = F_{11} \cdot x_1 + F_{12} \cdot y_1 \\ y_1^* = F_{21} \cdot x_1 + F_{22} \cdot y_1 \end{cases} \quad (4)$$

And

$$\begin{cases} x_2^* = F_{11} \cdot x_2 + F_{12} \cdot y_2 \\ y_2^* = F_{21} \cdot x_2 + F_{22} \cdot y_2 \end{cases} \quad (5)$$

After solving the system

$$\begin{cases} x_1^* = F_{11} \cdot x_1 + F_{12} \cdot y_1 \\ y_1^* = F_{21} \cdot x_1 + F_{22} \cdot y_1 \\ x_2^* = F_{11} \cdot x_2 + F_{12} \cdot y_2 \\ y_2^* = F_{21} \cdot x_2 + F_{22} \cdot y_2 \end{cases} \quad (6)$$

$$F_{11} = \frac{y_2 \cdot x_1^* - y_1 \cdot x_2^*}{x_1 \cdot y_2 - x_2 \cdot y_1} \quad (7)$$

$$F_{12} = \frac{-(x_2 \cdot x_1^*) + x_1 \cdot x_2^*}{x_1 \cdot y_2 - x_2 \cdot y_1} \quad (8)$$

$$F_{21} = \frac{y_2 \cdot y_1^* - y_1 \cdot y_2^*}{x_1 \cdot y_2 - x_2 \cdot y_1} \quad (9)$$

$$F_{22} = \frac{-(x_2 \cdot y_1^*) - x_1 \cdot y_2^*}{x_1 \cdot y_2 - x_2 \cdot y_1} \quad (10)$$

From the deformation gradient tensor  $[F]$  the Cauchy-Green tensor is defined as:

$$[C] = [F][F]^T \quad (11)$$

In contrast to  $[F]$  which can be non-symmetric  $[C]$  is a symmetric tensor such that:

$$C_{11} = F_{11}^2 + F_{12}^2 \quad (12)$$

$$C_{12} = C_{21} = F_{11} \cdot F_{21} + F_{12} \cdot F_{22} \quad (13)$$

$$C_{22} = F_{21}^2 + F_{22}^2 \quad (14)$$

The Green-Lagrange tensor is given from  $[C]$  by

$$[E] = \frac{1}{2} \cdot ([F][F]^T - [I]) \quad (15)$$

$$E_{11} = \frac{F_{11}^2 + F_{12}^2 - 1}{2} \quad (16)$$

$$E_{12} = \frac{F_{11} \cdot F_{12} + F_{22} \cdot F_{21}}{2} \quad (17)$$

$$E_{21} = \frac{F_{11} \cdot F_{12} + F_{22} \cdot F_{21}}{2} \quad (18)$$

$$E_{22} = \frac{F_{12}^2 + F_{22}^2 - 1}{2} \quad (19)$$

The principal deformations are given by:

$$\begin{cases} \varepsilon_I = \ln(E_{11} + E_{22} + \sqrt{(E_{11} + E_{22})^2 + (2 \cdot E_{12})^2}) + 1 \\ \varepsilon_{II} = \ln(E_{11} + E_{22} - \sqrt{(E_{11} + E_{22})^2 + (2 \cdot E_{12})^2}) + 1 \end{cases} \quad (20)$$

So deformations along the normal and tangent are represented

by the vector  $\begin{pmatrix} \varepsilon_I \\ \varepsilon_{II} \end{pmatrix}$ .

The deformations at the point  $x$  are represented by the norm of the deformation vector at point  $x$  (Euclidean distances).

**2.5.2 Centred finite differences**

Deformations are calculated from displacement measure at 3 and 5 points.

Centred finite differences at 3 points:

$$\varepsilon = \frac{U^{n+1} - U^{n-1}}{2s} \tag{21}$$

Centred finite differences at 5 points:

$$\varepsilon = \frac{-U^{n+2} + 8U^{n+1} - 8U^{n-1} + U^{n-2}}{12s} \tag{22}$$

Where  $s$  is a constant representing the difference in position of adjacent points, and  $U^i$  their displacement.

**3 THE EXPERIMENTAL RESULTS**

We present an experimental result to see the deformation achieved in a multi temporal image pair.

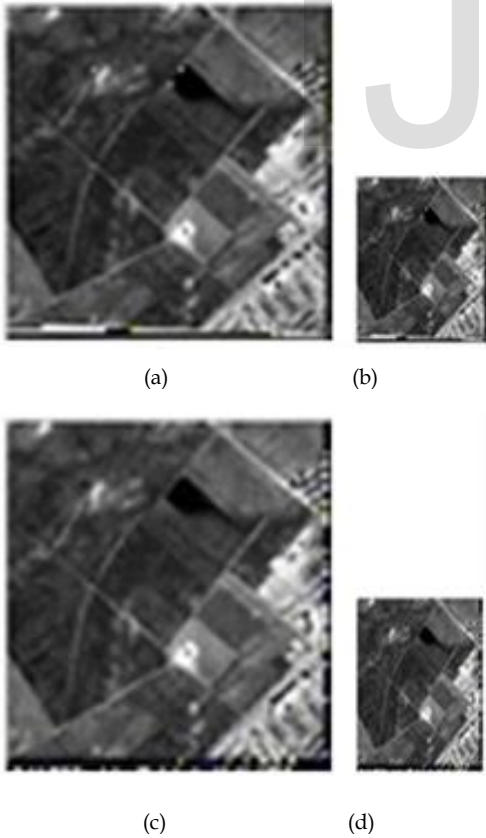


Fig. 4. The pair of original images (a,b), the pair of approximate images after the wavelet analysis (c,d).



Fig. 5. Disparity map using our method (a) with CCNC (b) with measurement of MORAVEC.

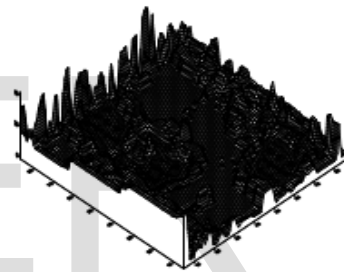
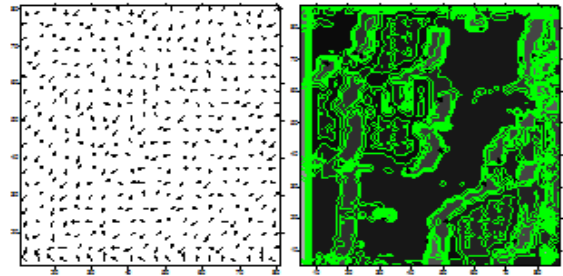


Fig. 6. Deformation field calculated by centred finite differences.

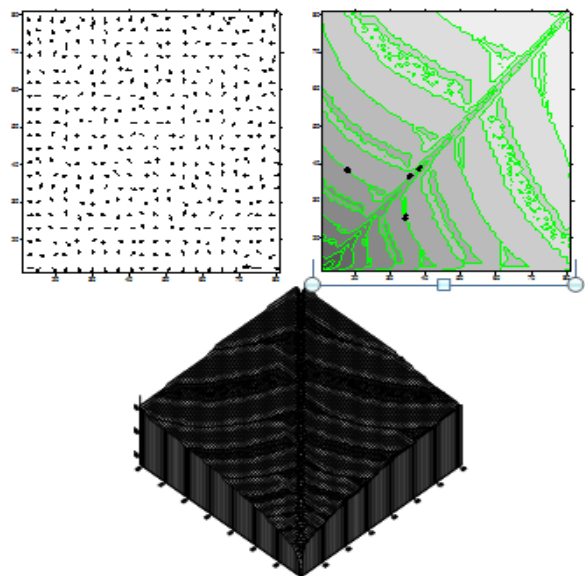


Fig.7. The deformation field calculated by the average strain of triangular elements of the localized structure.

